

THE MEASUREMENT OF ANGLES

- *from Babylonia (circa 1700 B.C.)
to England (circa 1714)*

IN A NUTSHELL

THE MEASUREMENT OF ANGLES

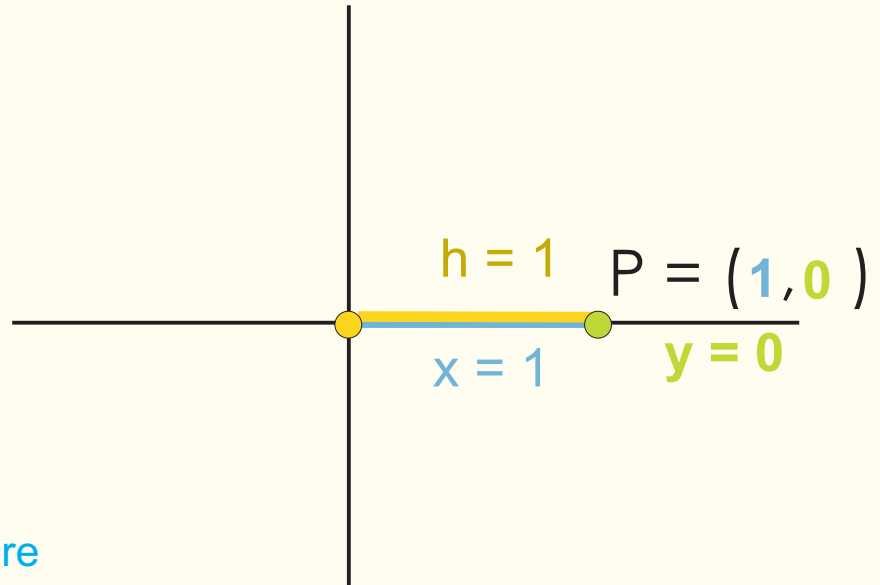
TRIGONOMETRIC MEASURE of 0°

$$\sin (0^\circ) := \frac{\text{opp}}{\text{hyp}} = \frac{0}{1} = 0$$

$$\cos (0^\circ) := \frac{\text{adj}}{\text{hyp}} = \frac{1}{1} = 1$$

$$\tan (0^\circ) := \frac{\text{opp}}{\text{adj}} = \frac{0}{1} = 0$$

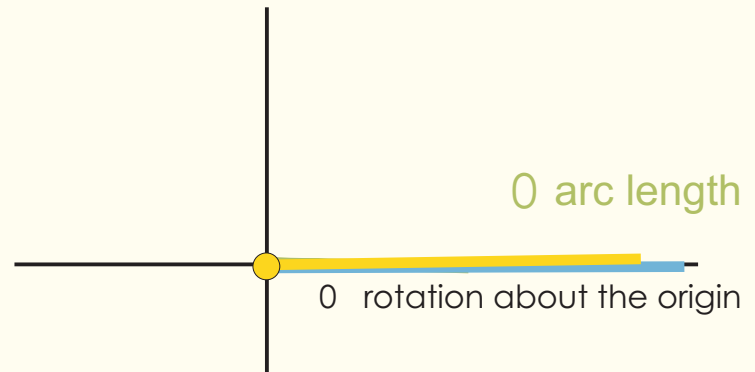
TRIANGULAR measure



DEGREE MEASURE of 0°

$$0^\circ = 0/360 \text{ of a full revolution}$$

ROTATIONAL measure



RADIAN MEASURE of 0°

$$0^\circ = 0 \text{ radians}$$

ARC LENGTH measure



THE MEASUREMENT OF ANGLES

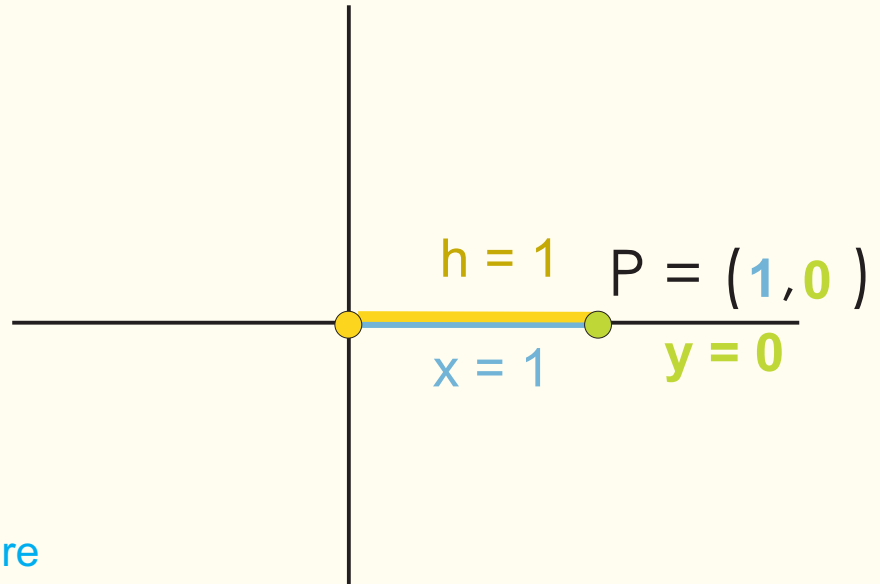
TRIGONOMETRIC MEASURE of 0°

$$\sin (0^\circ) := \frac{\text{opp}}{\text{hyp}} = \frac{0}{1} = 0$$

$$\cos (0^\circ) := \frac{\text{adj}}{\text{hyp}} = \frac{1}{1} = 1$$

$$\tan (0^\circ) := \frac{\text{opp}}{\text{adj}} = \frac{0}{1} = 0$$

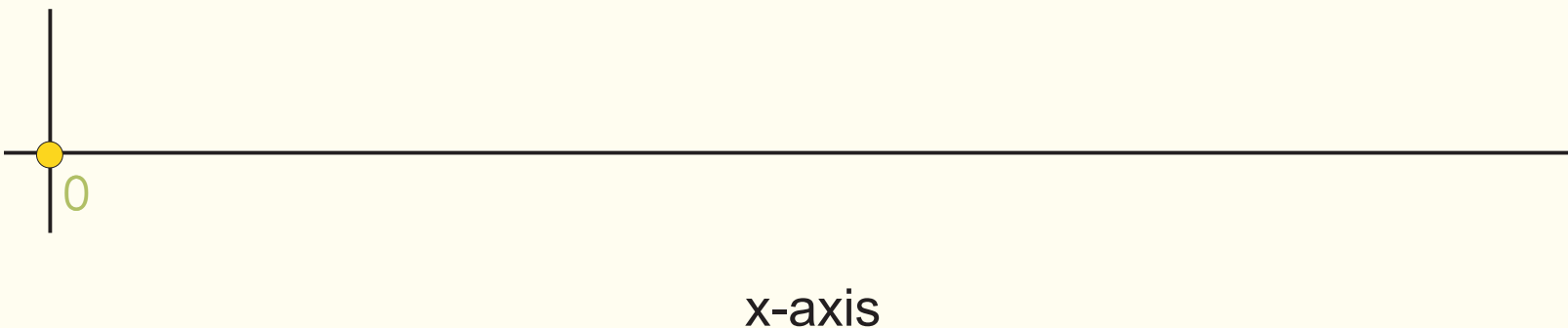
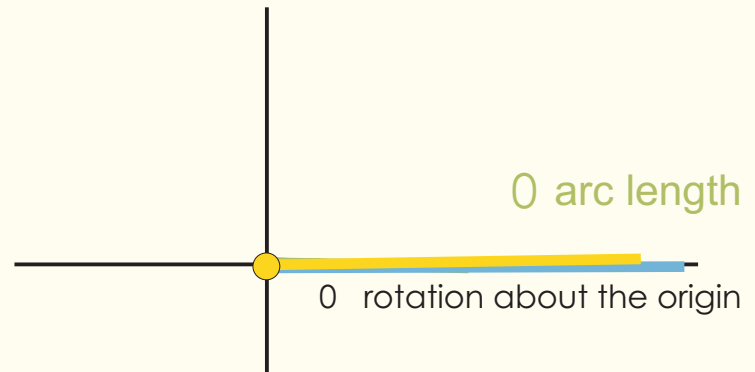
TRIANGULAR measure



RADIAN MEASURE of 0°

$$0^\circ = 0 \text{ radians}$$

as LINEAR measure



THE MEASUREMENT OF ANGLES

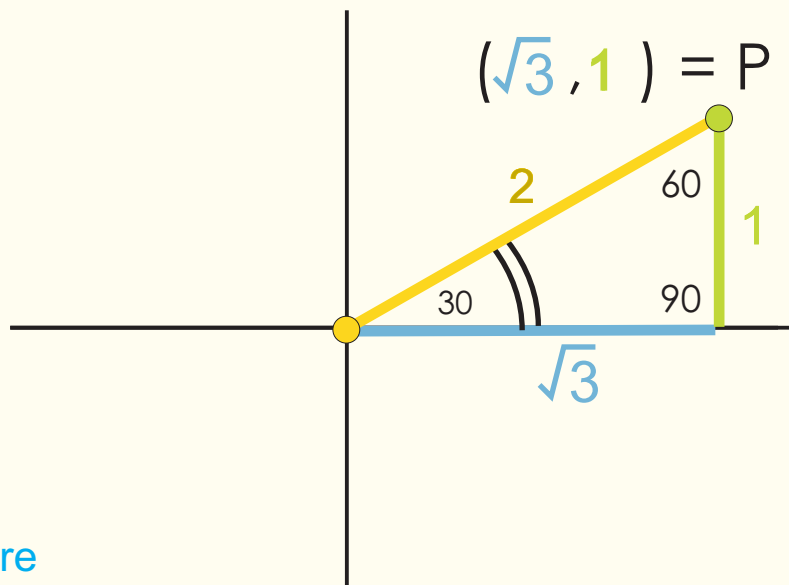
TRIGONOMETRIC MEASURE of 30°

$$\sin(30^\circ) := \frac{\text{opp}}{\text{hyp}} = \frac{1}{2}$$

$$\cos(30^\circ) := \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{3}}{2}$$

$$\tan(30^\circ) := \frac{\text{opp}}{\text{adj}} = \frac{1}{\sqrt{3}}$$

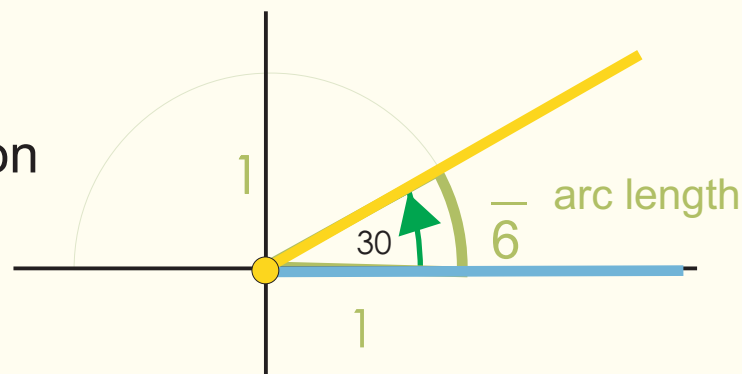
TRIANGULAR measure



DEGREE MEASURE of 30°

$30^\circ = 1/12$ of a full 360° revolution

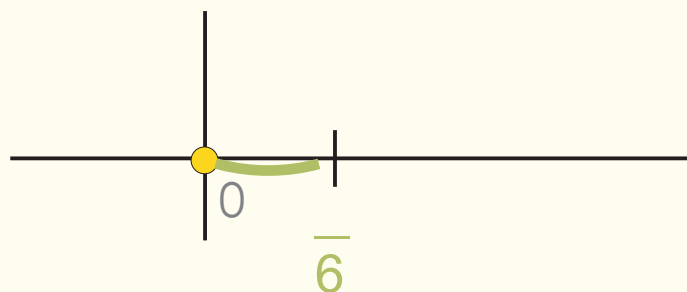
ROTATIONAL measure



RADIAN MEASURE of 30°

$$30^\circ = \frac{1}{6} \text{ radians}$$

ARC LENGTH measure



THE MEASUREMENT OF ANGLES

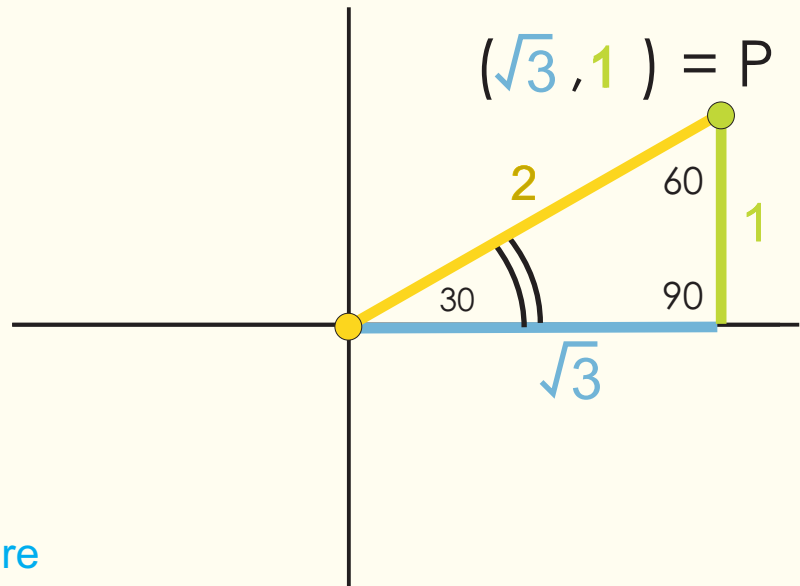
TRIGONOMETRIC MEASURE of 30°

$$\sin(30^\circ) := \frac{\text{opp}}{\text{hyp}} = \frac{1}{2}$$

$$\cos(30^\circ) := \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{3}}{2}$$

$$\tan(30^\circ) := \frac{\text{opp}}{\text{adj}} = \frac{1}{\sqrt{3}}$$

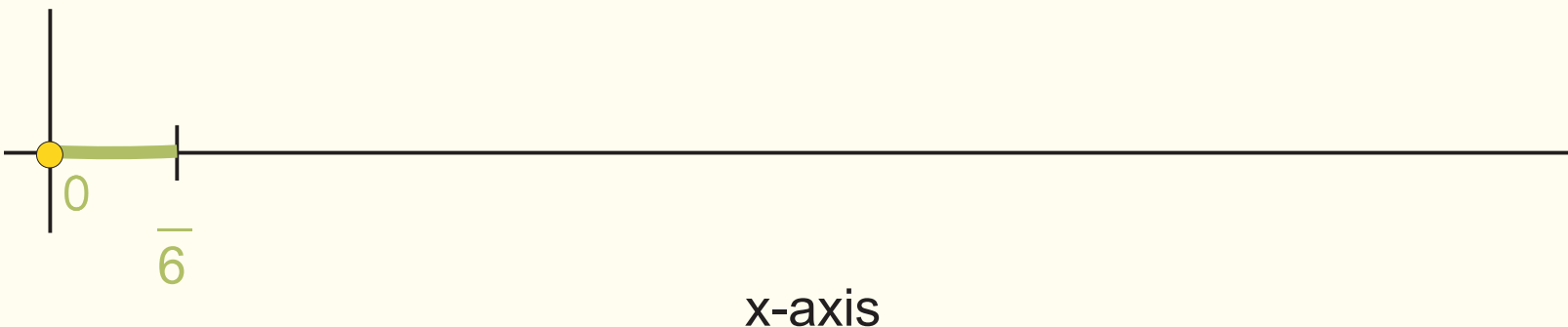
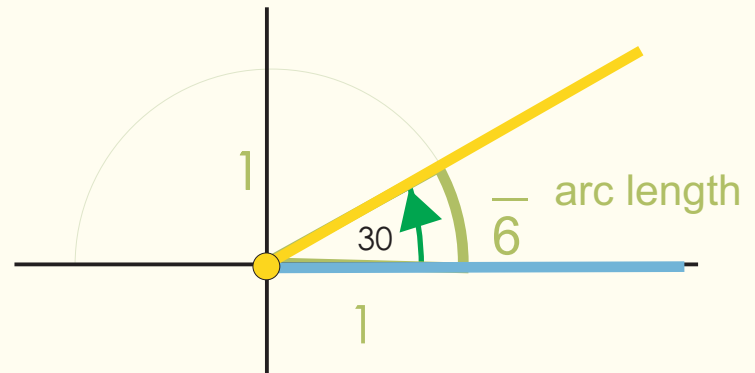
TRIANGULAR measure



RADIAN MEASURE of 30°

$$30^\circ = \frac{\pi}{6} \text{ radians}$$

as LINEAR measure



THE MEASUREMENT OF ANGLES

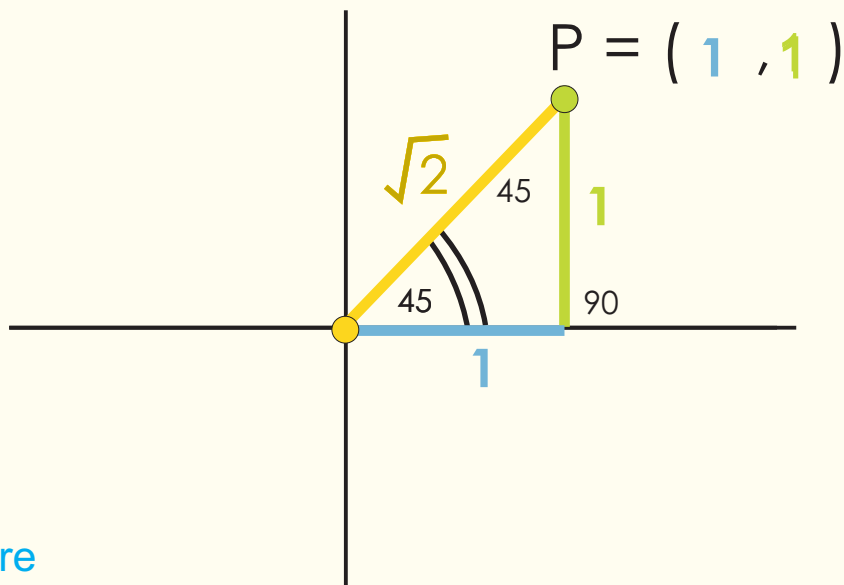
TRIGONOMETRIC MEASURE of 45°

$$\sin(45^\circ) := \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{2}}$$

$$\cos(45^\circ) := \frac{\text{adj}}{\text{hyp}} = \frac{1}{\sqrt{2}}$$

$$\tan(45^\circ) := \frac{\text{opp}}{\text{adj}} = \frac{1}{1}$$

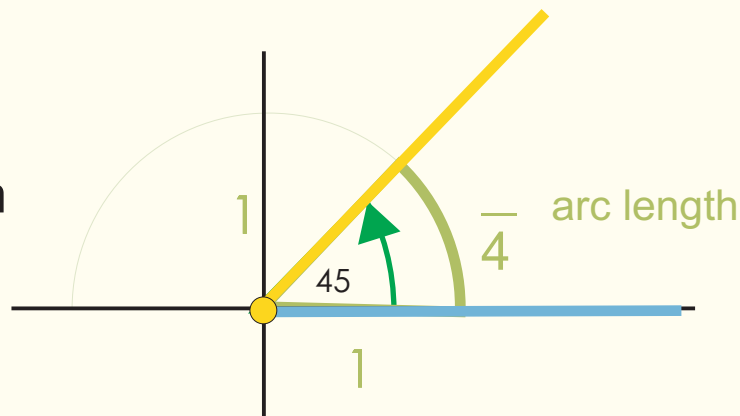
TRIANGULAR measure



DEGREE MEASURE of 45°

$45^\circ = 1/8$ of a full 360° revolution

ROTATIONAL measure



RADIAN MEASURE of 45°

$$45^\circ = \frac{1}{4} \text{ radians}$$

ARC LENGTH measure



THE MEASUREMENT OF ANGLES

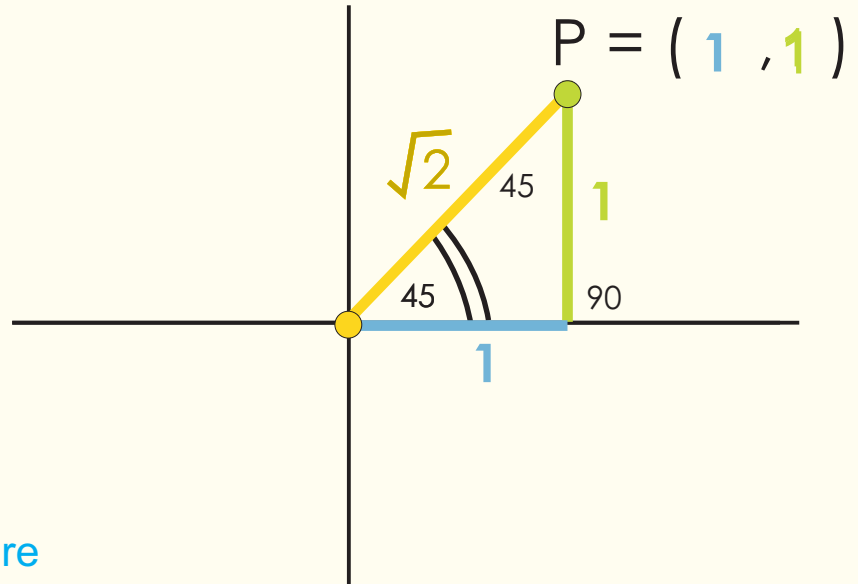
TRIGONOMETRIC MEASURE of 45°

$$\sin(45^\circ) := \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{2}}$$

$$\cos(45^\circ) := \frac{\text{adj}}{\text{hyp}} = \frac{1}{\sqrt{2}}$$

$$\tan(45^\circ) := \frac{\text{opp}}{\text{adj}} = \frac{1}{1}$$

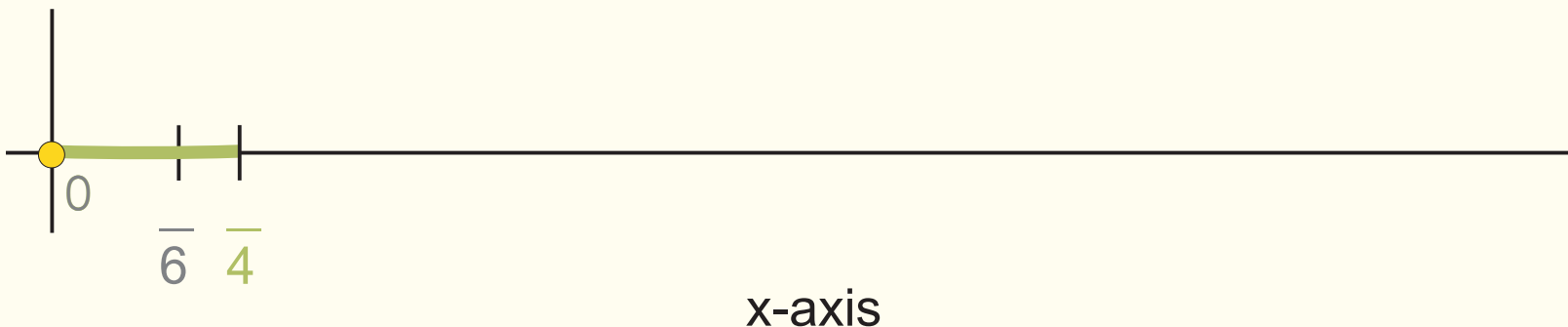
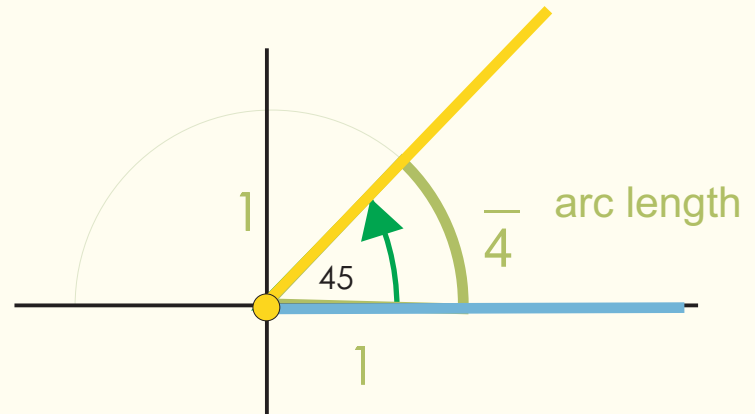
TRIANGULAR measure



RADIAN MEASURE of 45°

$$45^\circ = \frac{\pi}{4} \text{ radians}$$

as LINEAR measure



x-axis

THE MEASUREMENT OF ANGLES

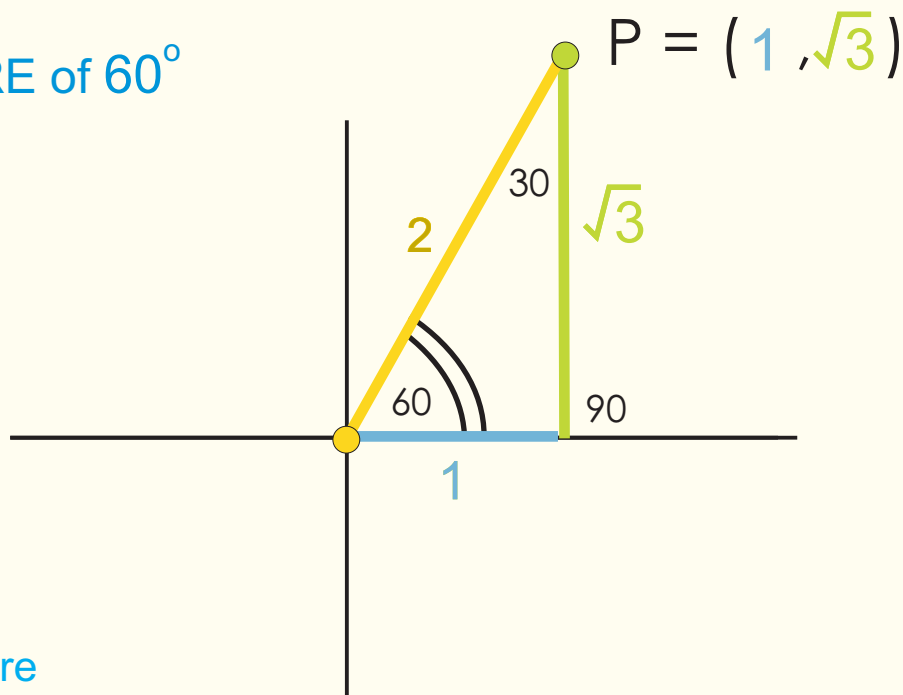
TRIGONOMETRIC MEASURE of 60°

$$\sin(60^\circ) := \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{3}}{2}$$

$$\cos(60^\circ) := \frac{\text{adj}}{\text{hyp}} = \frac{1}{2}$$

$$\tan(60^\circ) := \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{3}}{1}$$

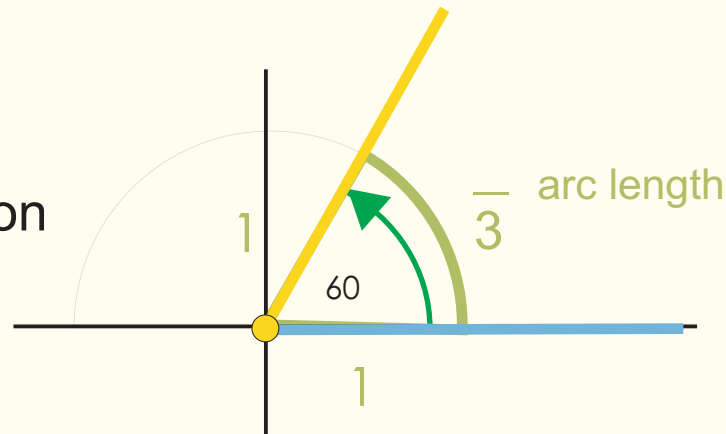
TRIANGULAR measure



DEGREE MEASURE of 60°

$60^\circ = 1/6$ of a full 360° revolution

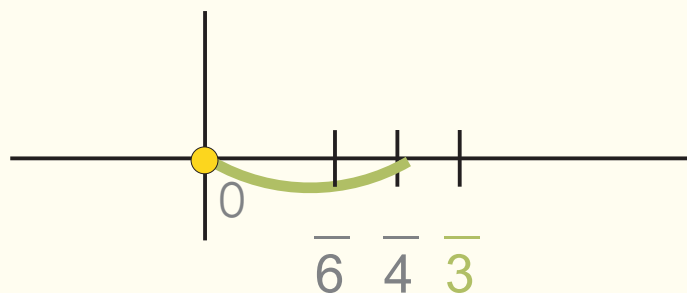
ROTATIONAL measure



RADIAN MEASURE of 60°

$$60^\circ = \frac{1}{3} \text{ radians}$$

ARC LENGTH measure



THE MEASUREMENT OF ANGLES

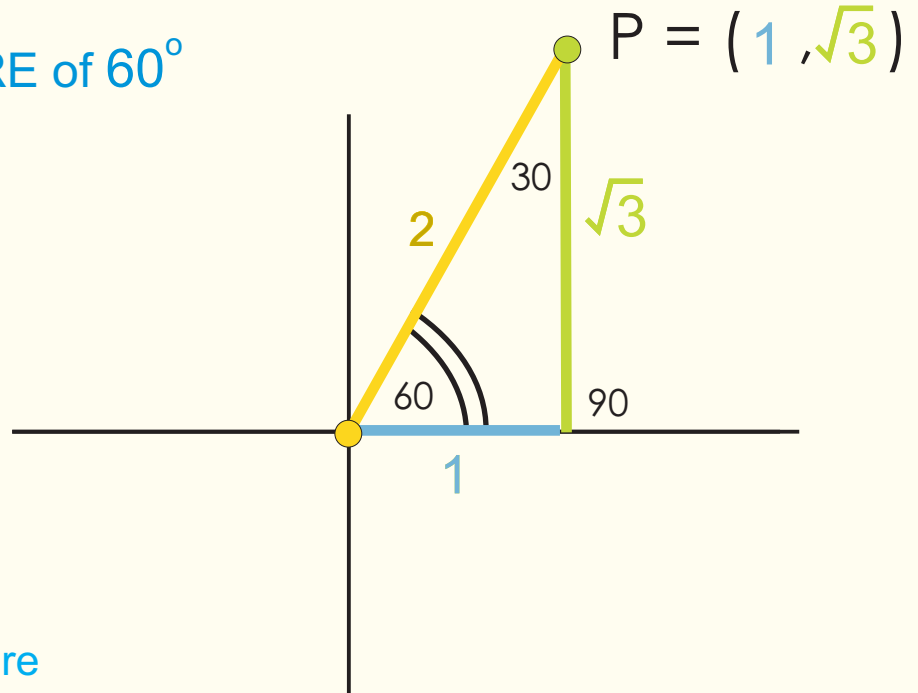
TRIGONOMETRIC MEASURE of 60°

$$\sin(60^\circ) := \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{3}}{2}$$

$$\cos(60^\circ) := \frac{\text{adj}}{\text{hyp}} = \frac{1}{2}$$

$$\tan(60^\circ) := \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{3}}{1}$$

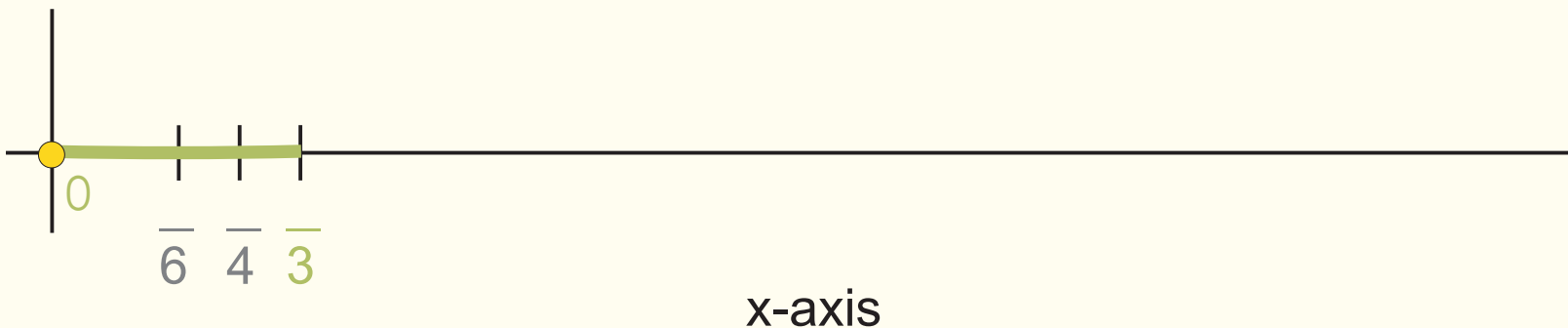
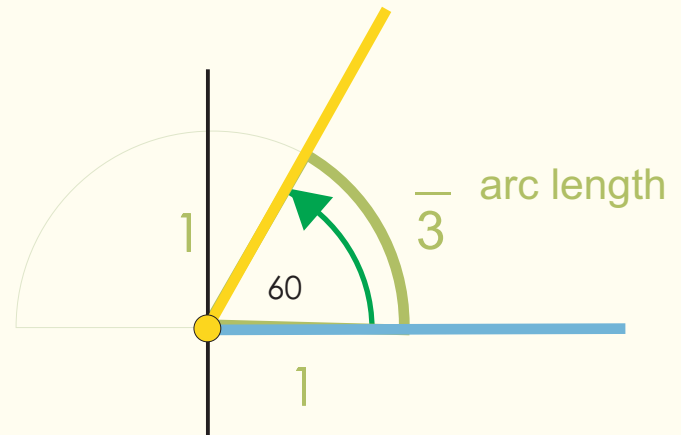
TRIANGULAR measure



RADIAN MEASURE of 60°

$$60^\circ = \frac{\pi}{3} \text{ radians}$$

as LINEAR measure



THE MEASUREMENT OF ANGLES

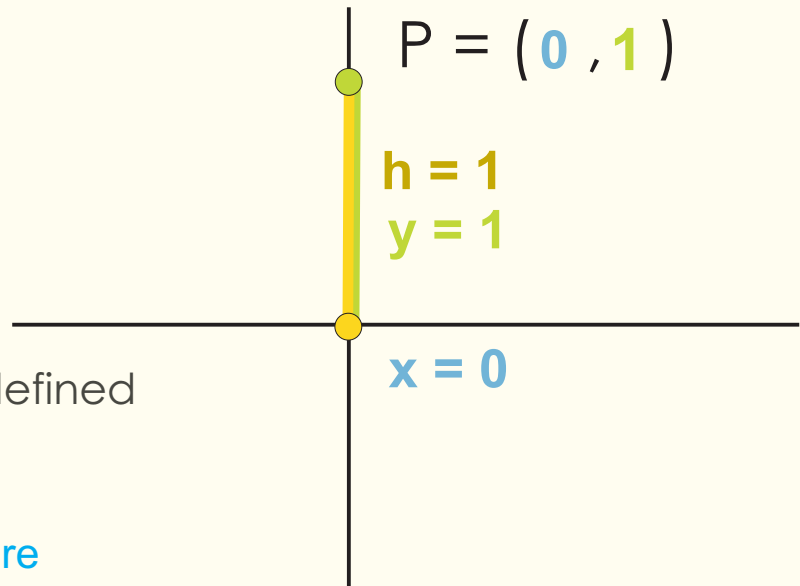
TRIGONOMETRIC MEASURE of 90°

$$\sin(90^\circ) := \frac{\text{opp}}{\text{hyp}} = \frac{1}{1} = 1$$

$$\cos(90^\circ) := \frac{\text{adj}}{\text{hyp}} = \frac{0}{1} = 0$$

$$\tan(90^\circ) := \frac{\text{opp}}{\text{adj}} = \frac{1}{0} \text{ undefined}$$

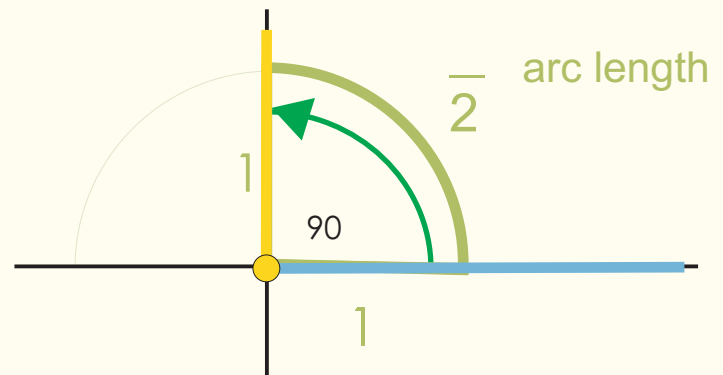
TRIANGULAR measure



DEGREE MEASURE of 90°

$90^\circ = 1/4$ of a full revolution

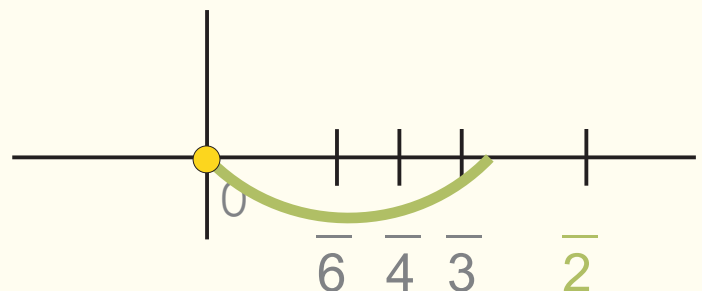
ROTATIONAL measure



RADIAN MEASURE of 90°

$$90^\circ = \frac{\pi}{2} \text{ radians}$$

ARC LENGTH measure



THE MEASUREMENT OF ANGLES

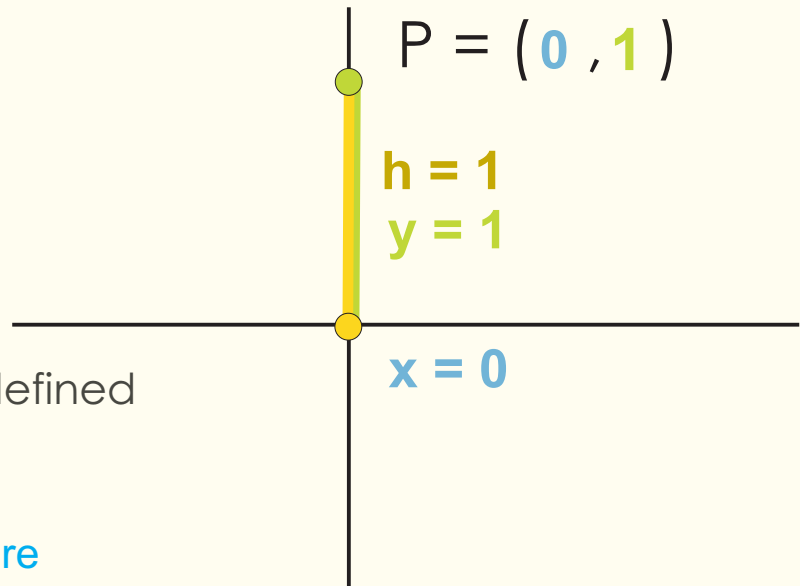
TRIGONOMETRIC MEASURE of 90°

$$\sin(90^\circ) := \frac{\text{opp}}{\text{hyp}} = \frac{1}{1} = 1$$

$$\cos(90^\circ) := \frac{\text{adj}}{\text{hyp}} = \frac{0}{1} = 0$$

$$\tan(90^\circ) := \frac{\text{opp}}{\text{adj}} = \frac{1}{0} \text{ undefined}$$

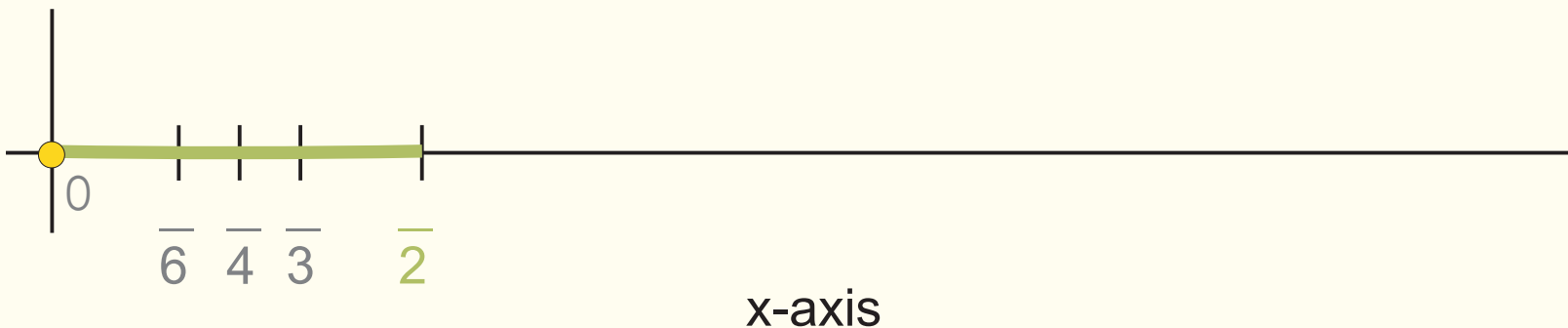
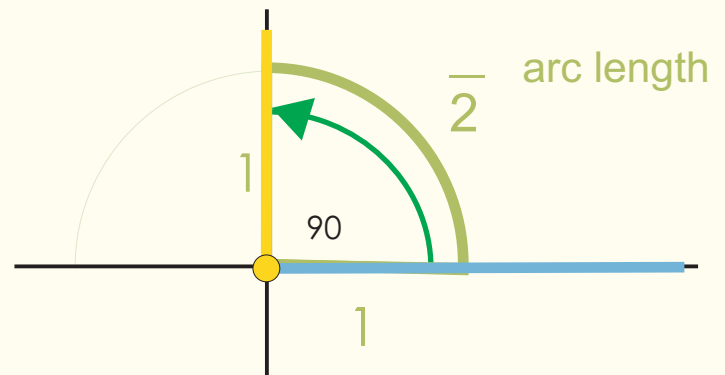
TRIANGULAR measure



RADIAN MEASURE of 90°

$$90^\circ = \frac{\pi}{2} \text{ radians}$$

as LINEAR measure



THE MEASUREMENT OF ANGLES

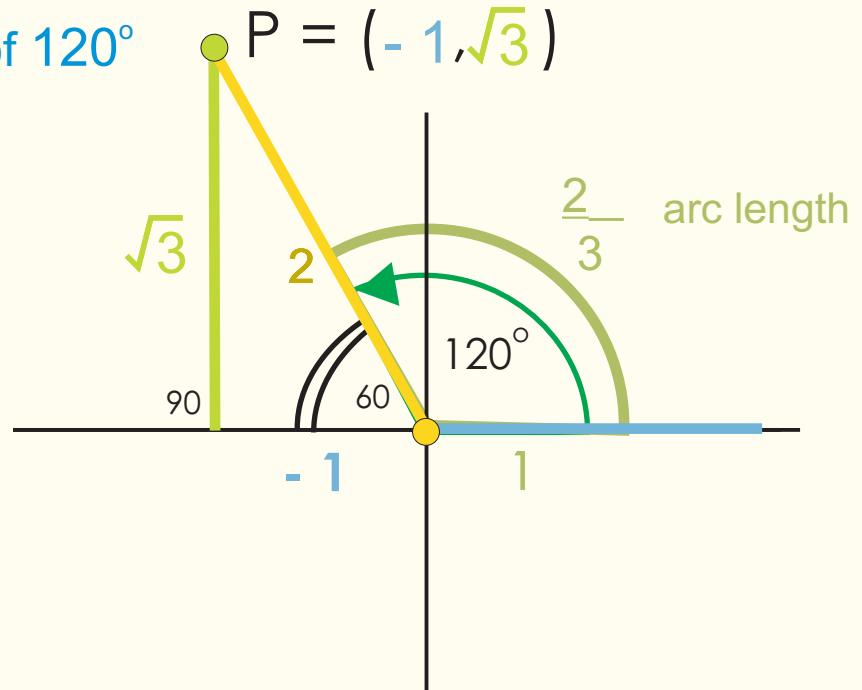
TRIGONOMETRIC MEASURE of 120°

$$\sin(120^\circ) := \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{3}}{2}$$

$$\cos(120^\circ) := \frac{\text{adj}}{\text{hyp}} = \frac{-1}{2}$$

$$\tan(120^\circ) := \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{3}}{-1}$$

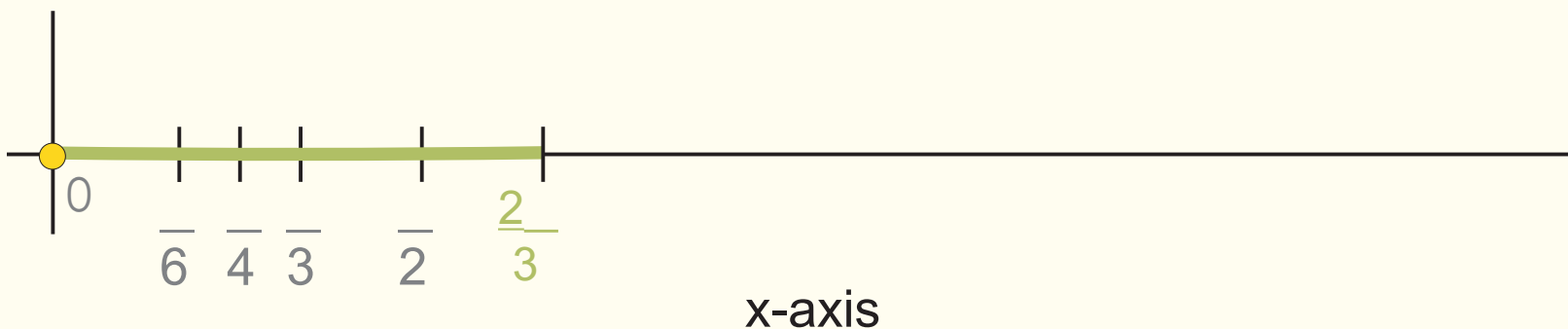
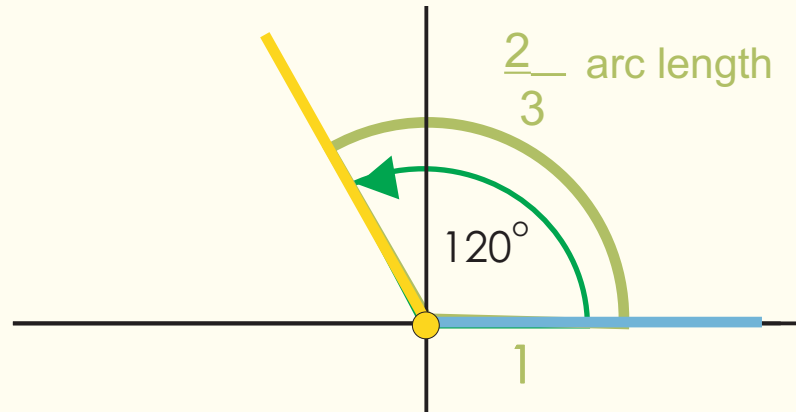
TRIANGULAR measure



RADIAN MEASURE of 120°

$$120^\circ = \frac{2}{3} \text{ radians}$$

as LINEAR measure



THE MEASUREMENT OF ANGLES

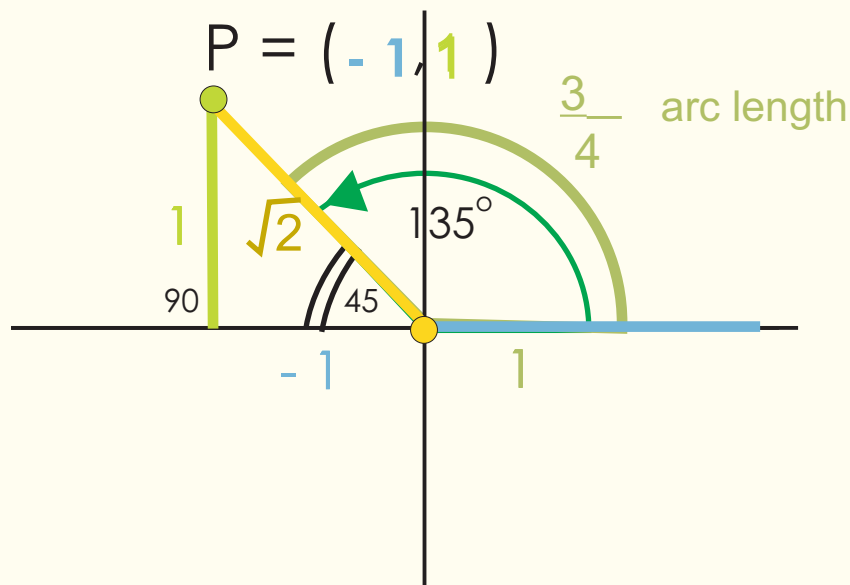
TRIGONOMETRIC MEASURE of 135°

$$\sin (135^\circ) := \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{2}}$$

$$\cos (135^\circ) := \frac{\text{adj}}{\text{hyp}} = \frac{-1}{\sqrt{2}}$$

$$\tan (135^\circ) := \frac{\text{opp}}{\text{adj}} = \frac{1}{-1}$$

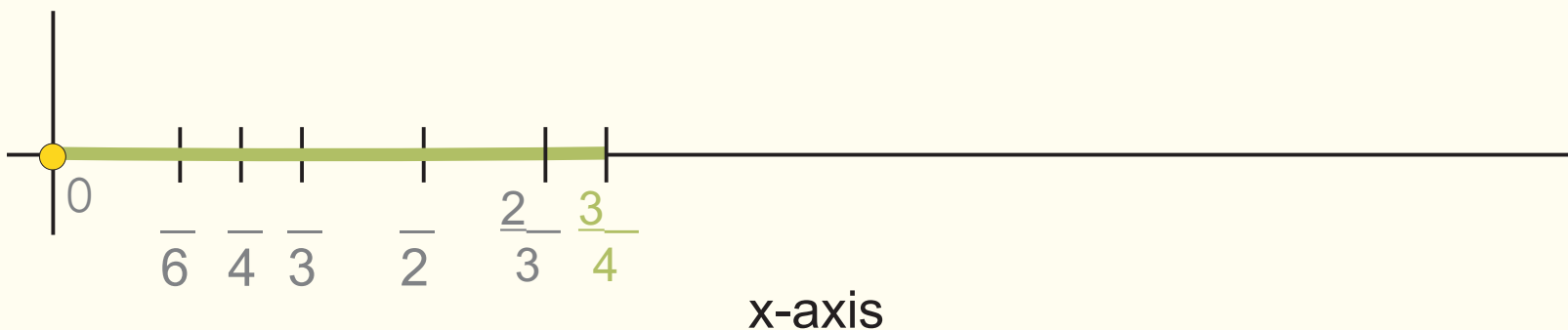
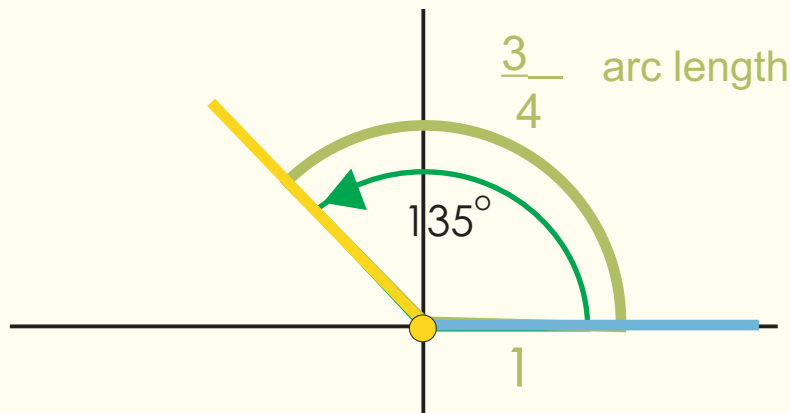
TRIANGULAR measure



RADIAN MEASURE of 135°

$$135^\circ = \frac{\quad}{4} \text{ radians}$$

as LINEAR measure



THE MEASUREMENT OF ANGLES

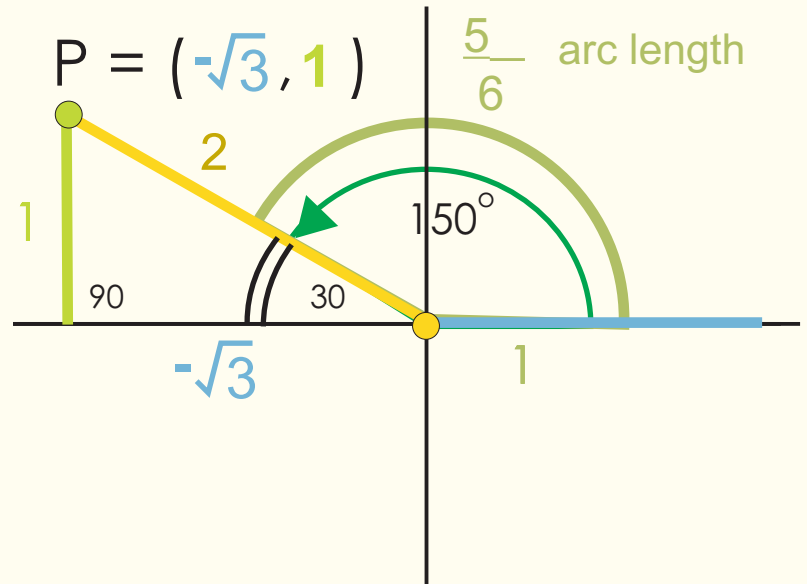
TRIGONOMETRIC MEASURE of 150°

$$\sin(150^\circ) := \frac{\text{opp}}{\text{hyp}} = \frac{1}{2}$$

$$\cos(150^\circ) := \frac{\text{adj}}{\text{hyp}} = \frac{-\sqrt{3}}{2}$$

$$\tan(150^\circ) := \frac{\text{opp}}{\text{adj}} = \frac{1}{-\sqrt{3}}$$

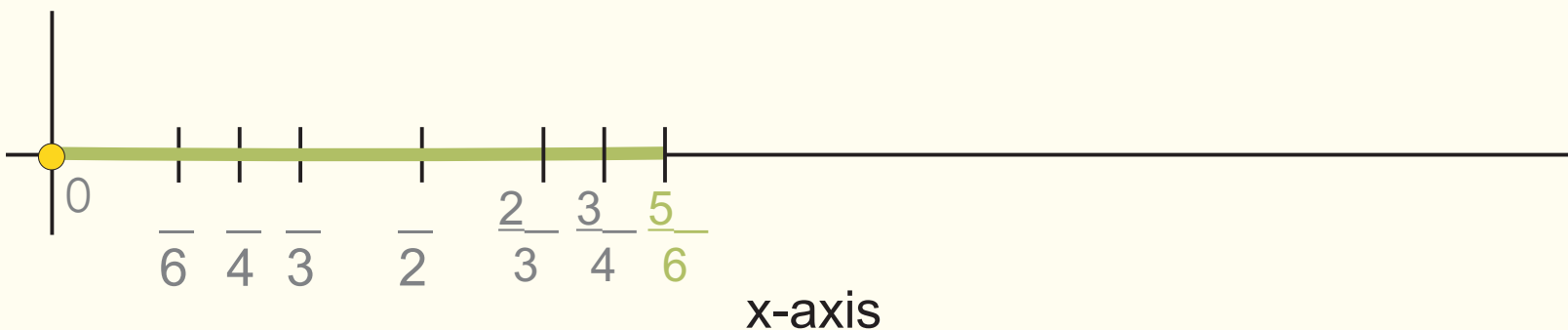
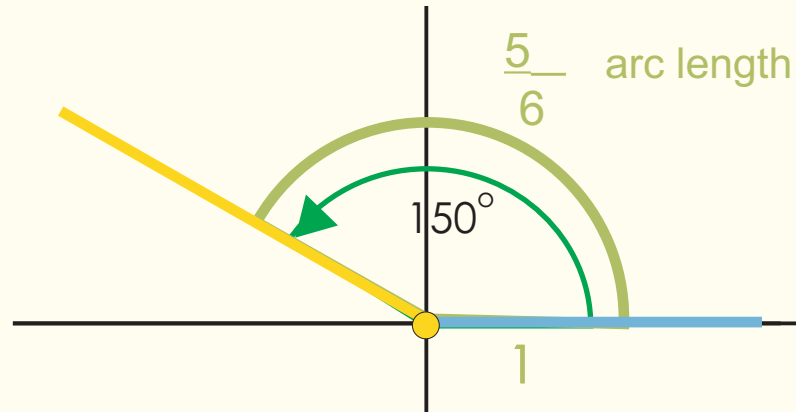
TRIANGULAR measure



RADIAN MEASURE of 150°

$$150^\circ = \frac{5}{6} \text{ radians}$$

as LINEAR measure



THE MEASUREMENT OF ANGLES

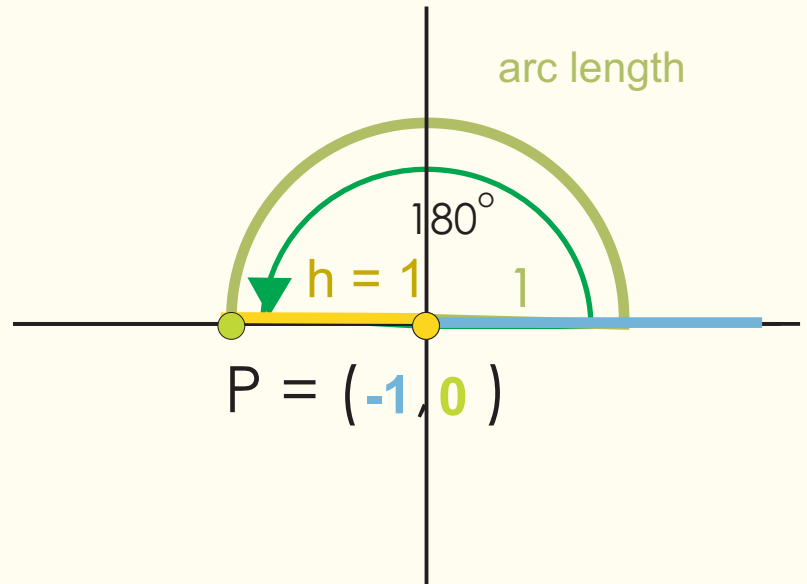
TRIGONOMETRIC MEASURE of 180°

$$\sin(180^\circ) := \frac{\text{opp}}{\text{hyp}} = \frac{0}{1} = 0$$

$$\cos(180^\circ) := \frac{\text{adj}}{\text{hyp}} = \frac{-1}{1} = -1$$

$$\tan(180^\circ) := \frac{\text{opp}}{\text{adj}} = \frac{0}{-1} = 0$$

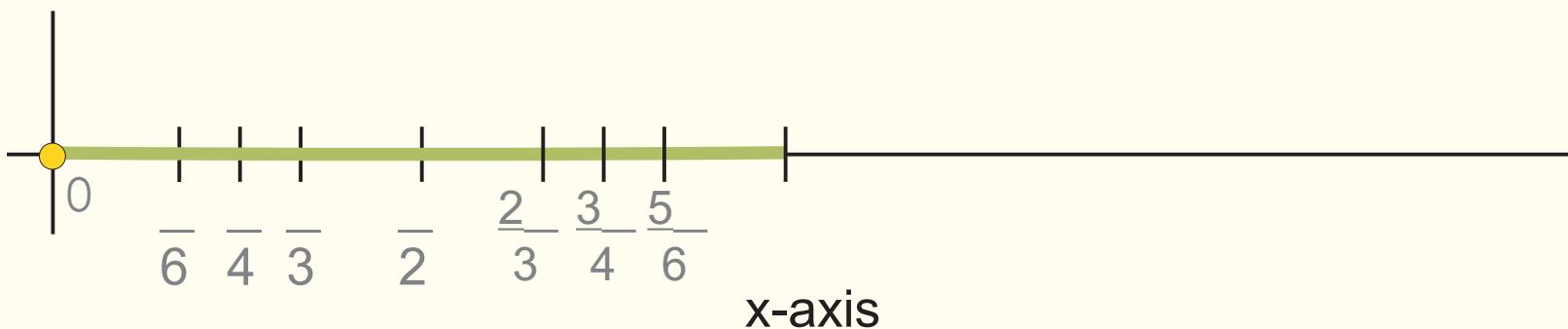
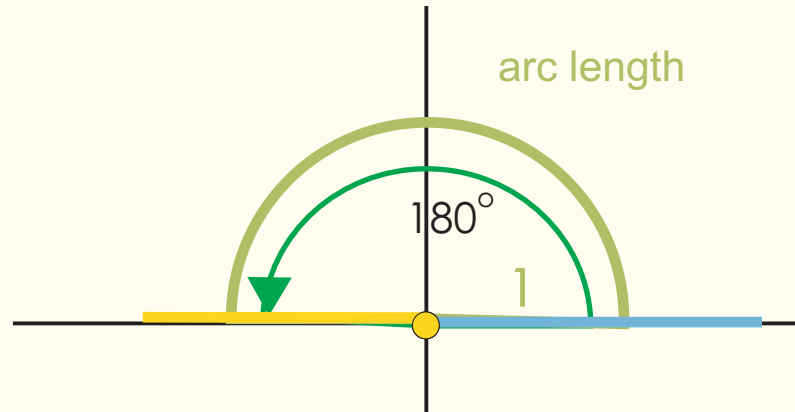
TRIANGULAR measure



RADIAN MEASURE of 180°

$$180^\circ = \text{radians}$$

as LINEAR measure



THE MEASUREMENT OF ANGLES

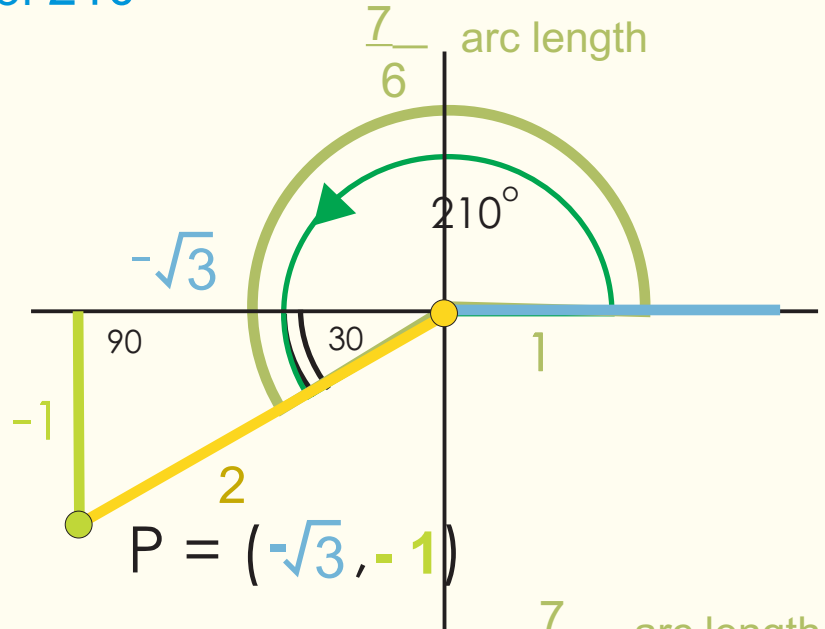
TRIGONOMETRIC MEASURE of 210°

$$\sin (210^\circ) := \frac{\text{opp}}{\text{hyp}} = \frac{-1}{2}$$

$$\cos (210^\circ) := \frac{\text{adj}}{\text{hyp}} = \frac{-\sqrt{3}}{2}$$

$$\tan (210^\circ) := \frac{\text{opp}}{\text{adj}} = \frac{-1}{-\sqrt{3}}$$

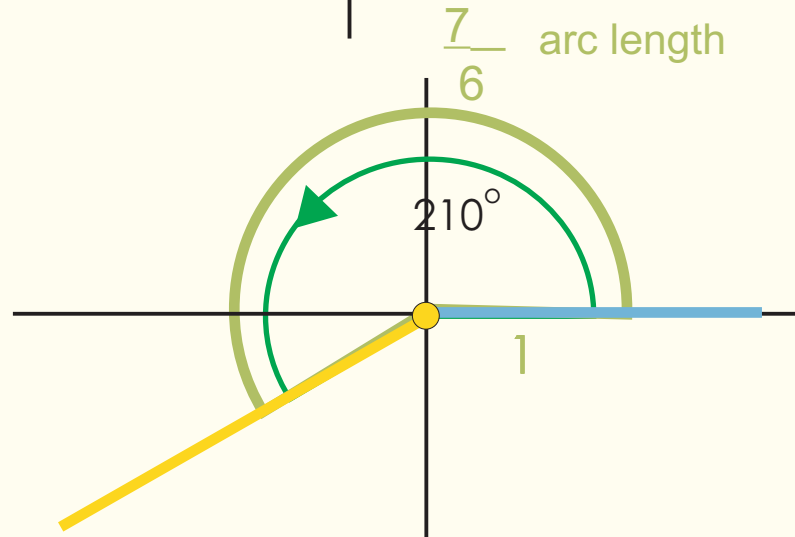
TRIANGULAR measure



RADIAN MEASURE of 210°

$$210^\circ = \frac{7}{6} \text{ radians}$$

as LINEAR measure



THE MEASUREMENT OF ANGLES

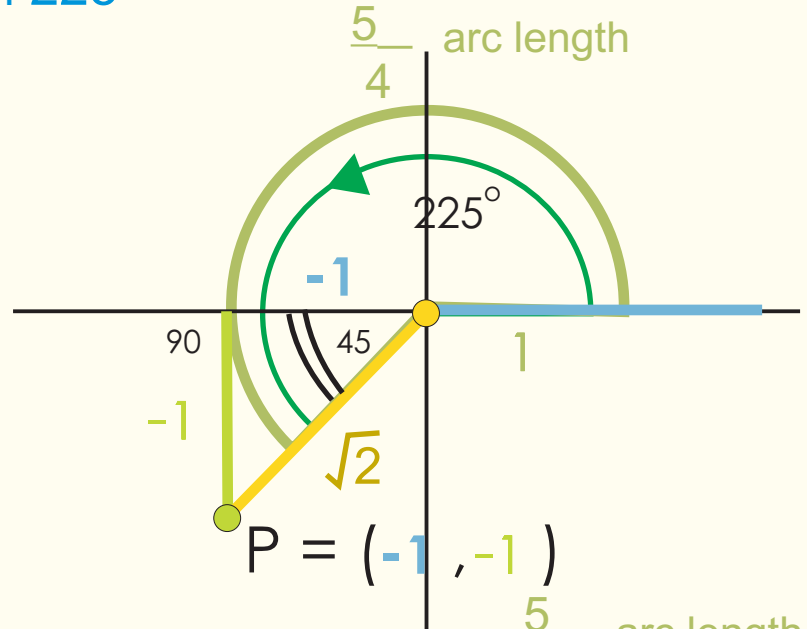
TRIGONOMETRIC MEASURE of 225°

$$\sin (225^\circ) := \frac{\text{opp}}{\text{hyp}} = \frac{-1}{\sqrt{2}}$$

$$\cos (225^\circ) := \frac{\text{adj}}{\text{hyp}} = \frac{-1}{\sqrt{2}}$$

$$\tan (225^\circ) := \frac{\text{opp}}{\text{adj}} = \frac{-1}{-1}$$

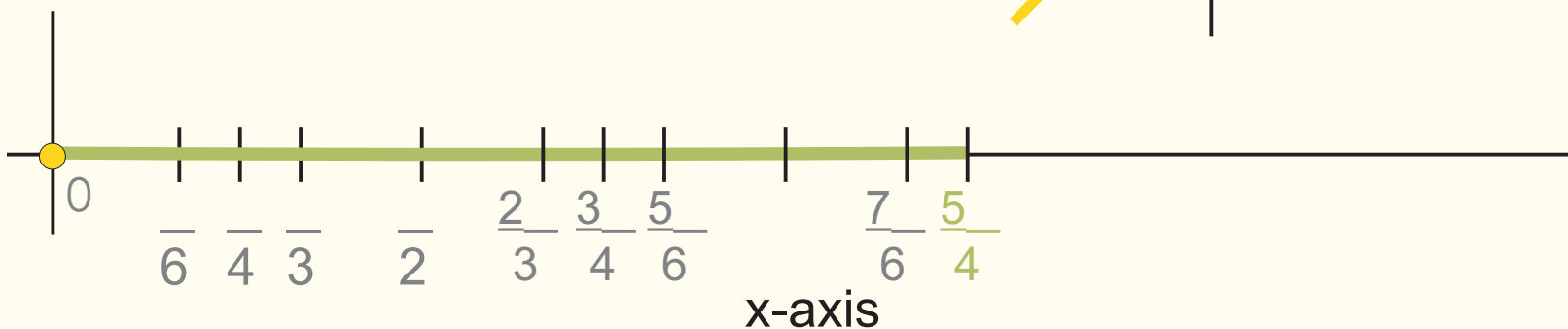
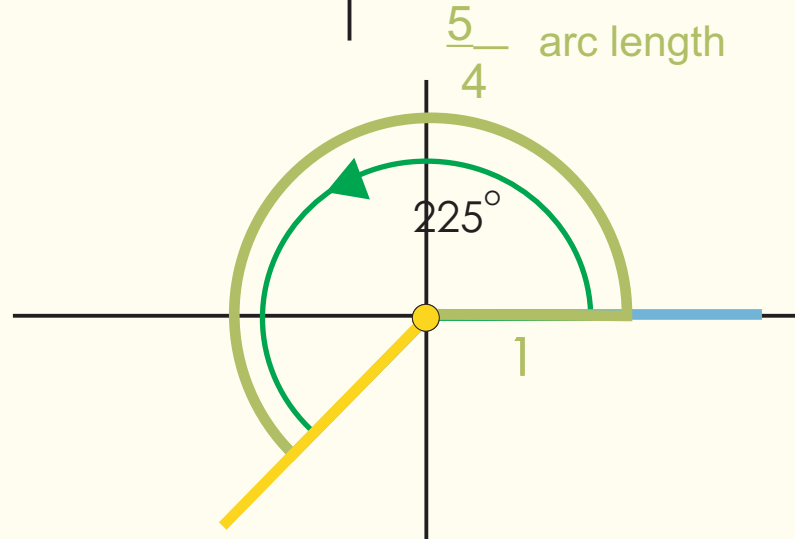
TRIANGULAR measure



RADIAN MEASURE of 225°

$$225^\circ = \frac{\quad}{4} \text{ radians}$$

as LINEAR measure



THE MEASUREMENT OF ANGLES

TRIGONOMETRIC MEASURE of 240°

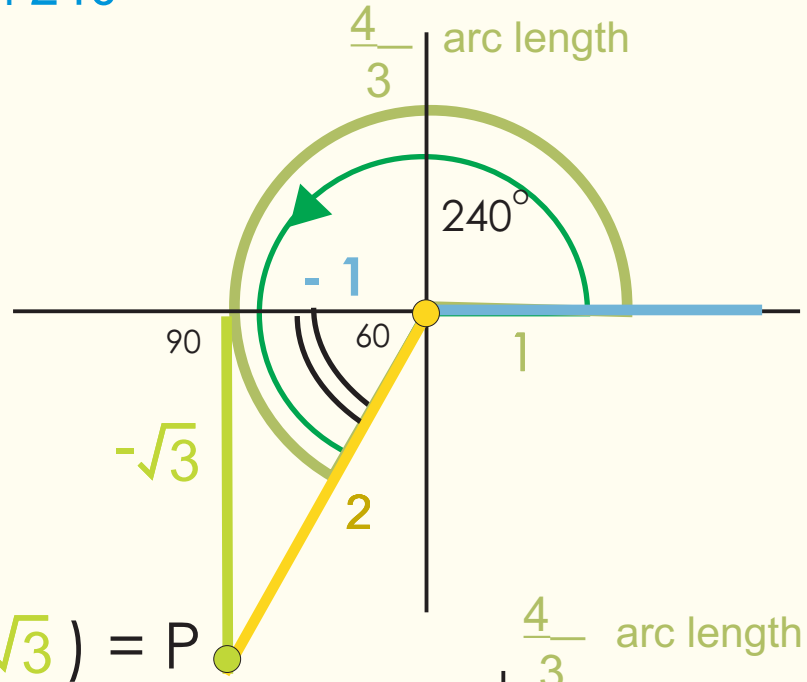
$$\sin (240^\circ) := \frac{\text{opp}}{\text{hyp}} = \frac{-\sqrt{3}}{2}$$

$$\cos (240^\circ) := \frac{\text{adj}}{\text{hyp}} = \frac{-1}{2}$$

$$\tan (240^\circ) := \frac{\text{opp}}{\text{adj}} = \frac{-\sqrt{3}}{-1}$$

TRIANGULAR measure

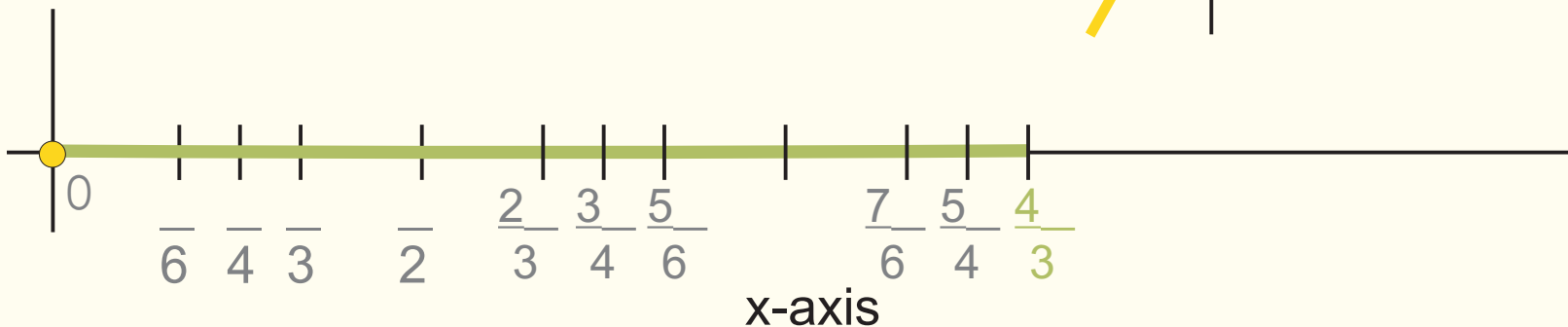
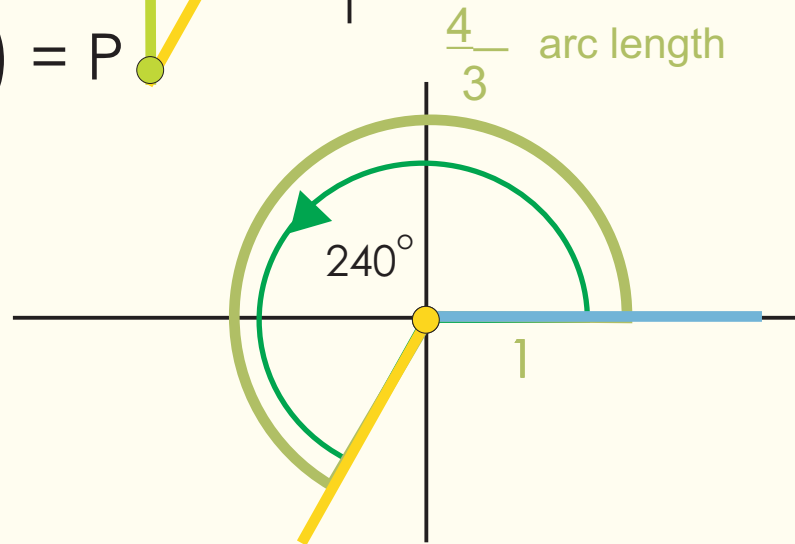
$$(-1, -\sqrt{3}) = P$$



RADIAN MEASURE of 240°

$$240^\circ = \frac{\quad}{3} \text{ radians}$$

as LINEAR measure



THE MEASUREMENT OF ANGLES

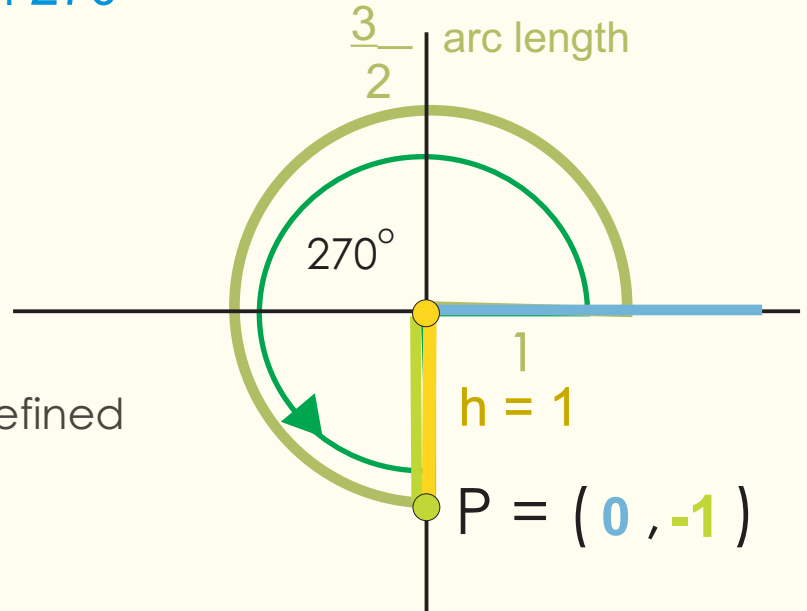
TRIGONOMETRIC MEASURE of 270°

$$\sin (270^\circ) := \frac{\text{opp}}{\text{hyp}} = \frac{-1}{1}$$

$$\cos (270^\circ) := \frac{\text{adj}}{\text{hyp}} = \frac{0}{1}$$

$$\tan (270^\circ) := \frac{\text{opp}}{\text{adj}} = \frac{-1}{0} \text{ undefined}$$

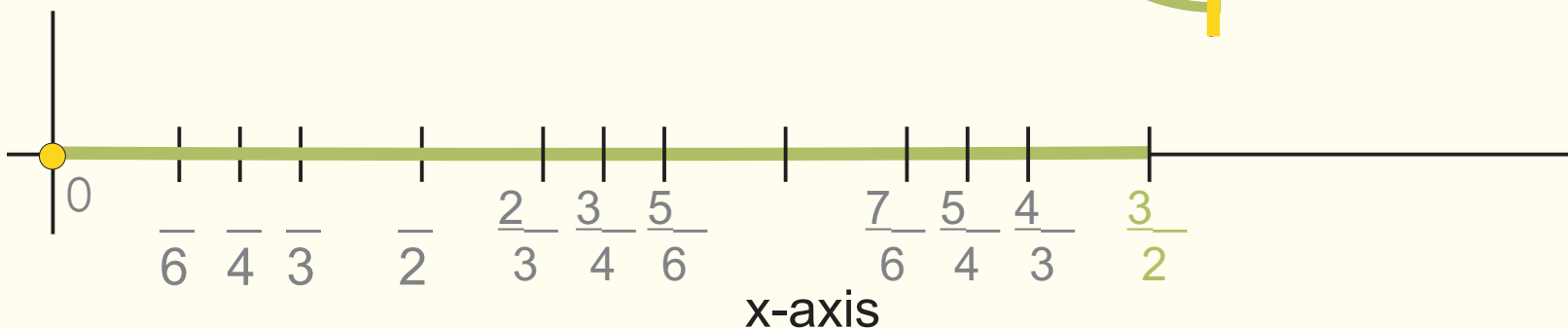
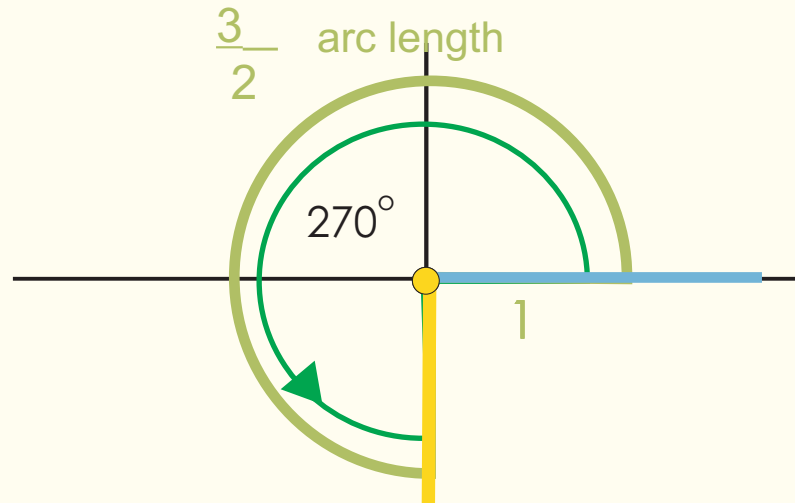
TRIANGULAR measure



RADIAN MEASURE of 270°

$$270^\circ = \frac{\quad}{2} \text{ radians}$$

as LINEAR measure



THE MEASUREMENT OF ANGLES

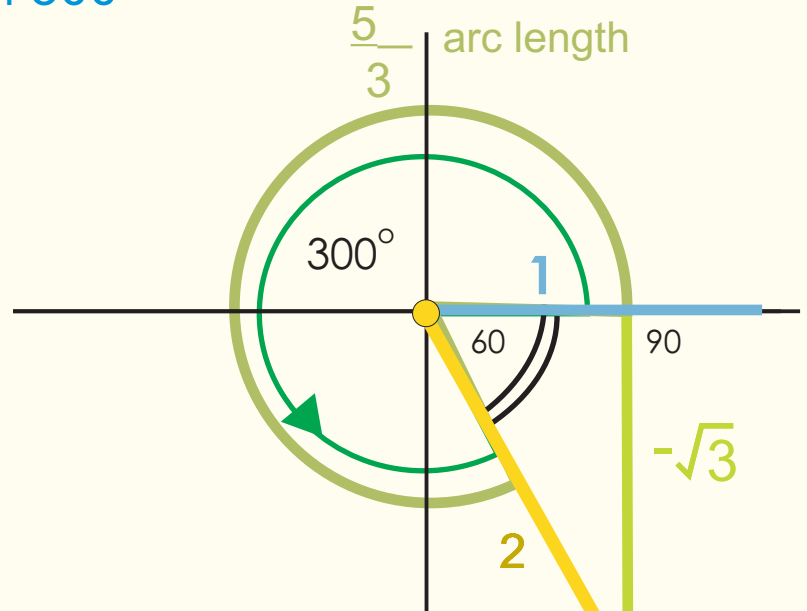
TRIGONOMETRIC MEASURE of 300°

$$\sin(300^\circ) := \frac{\text{opp}}{\text{hyp}} = \frac{-\sqrt{3}}{2}$$

$$\cos(300^\circ) := \frac{\text{adj}}{\text{hyp}} = \frac{1}{2}$$

$$\tan(300^\circ) := \frac{\text{opp}}{\text{adj}} = \frac{-\sqrt{3}}{1}$$

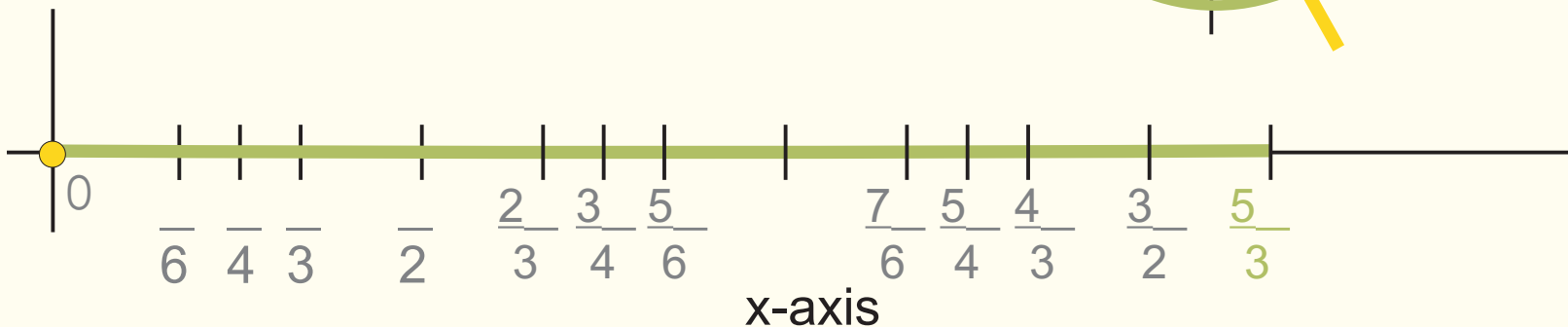
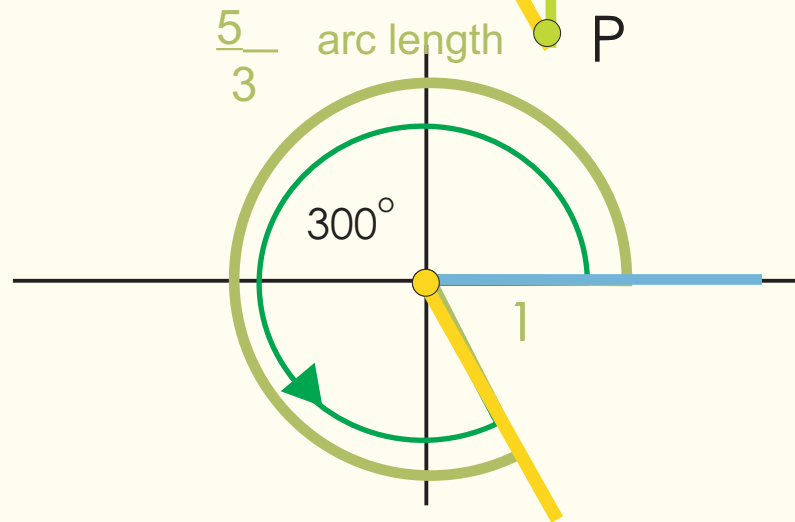
TRIANGULAR measure



RADIAN MEASURE of 300°

$$300^\circ = \frac{\quad}{3} \text{ radians}$$

as LINEAR measure



THE MEASUREMENT OF ANGLES

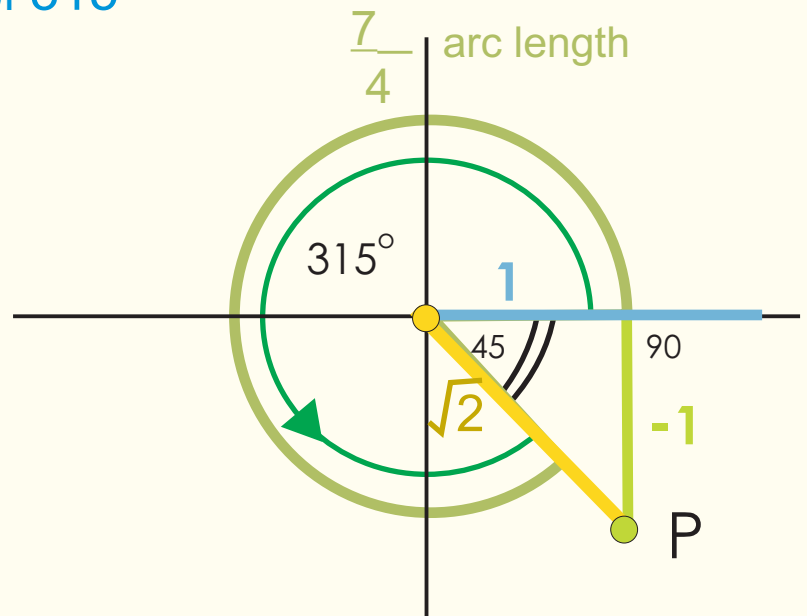
TRIGONOMETRIC MEASURE of 315°

$$\sin(315^\circ) := \frac{\text{opp}}{\text{hyp}} = \frac{-1}{\sqrt{2}}$$

$$\cos(315^\circ) := \frac{\text{adj}}{\text{hyp}} = \frac{1}{\sqrt{2}}$$

$$\tan(315^\circ) := \frac{\text{opp}}{\text{adj}} = \frac{-1}{1}$$

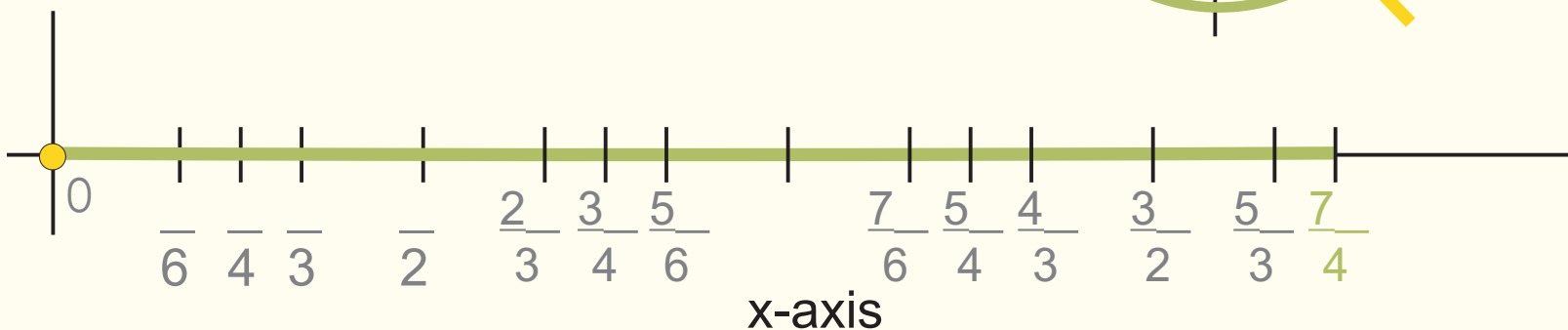
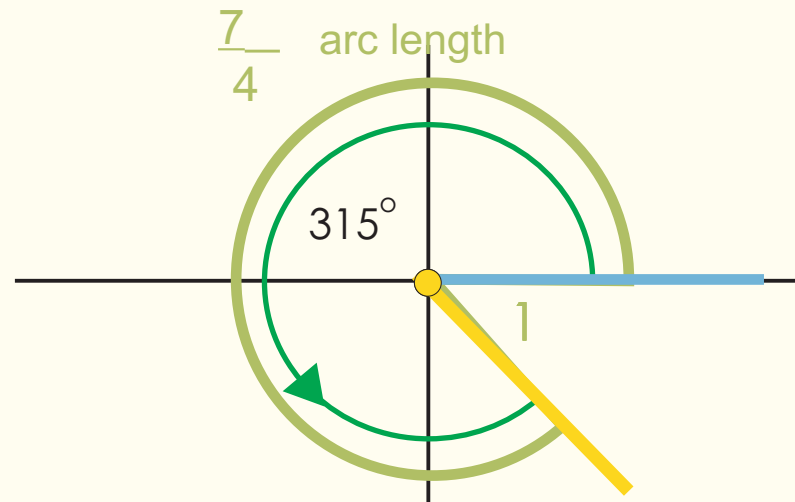
TRIANGULAR measure



RADIAN MEASURE of 315°

$$315^\circ = \frac{\quad}{4} \text{ radians}$$

as LINEAR measure



THE MEASUREMENT OF ANGLES

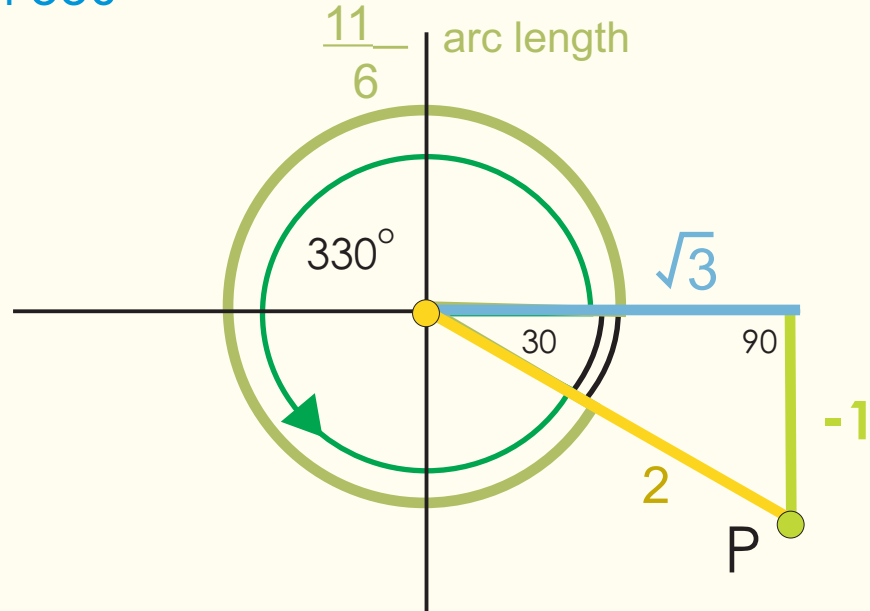
TRIGONOMETRIC MEASURE of 330°

$$\sin(330^\circ) := \frac{\text{opp}}{\text{hyp}} = \frac{-1}{2}$$

$$\cos(330^\circ) := \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{3}}{2}$$

$$\tan(330^\circ) := \frac{\text{opp}}{\text{adj}} = \frac{-1}{\sqrt{3}}$$

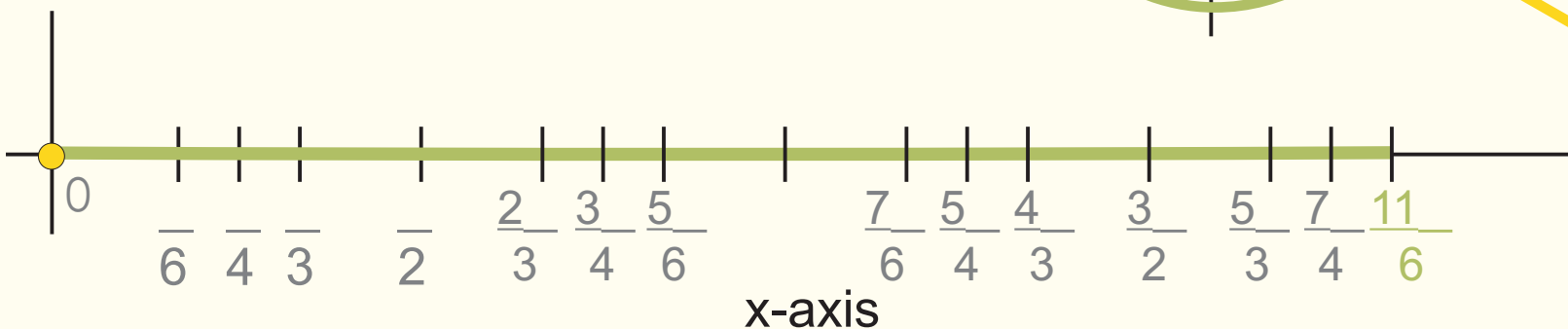
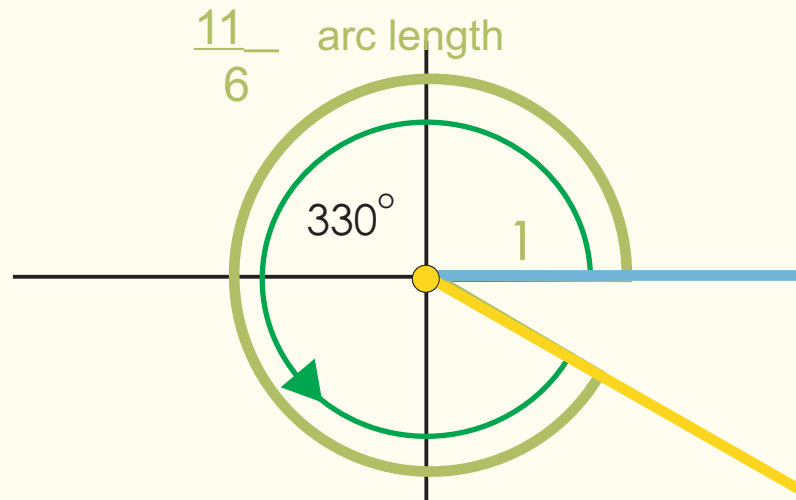
TRIANGULAR measure



RADIAN MEASURE of 330°

$$330^\circ = \frac{\quad}{6} \text{ radians}$$

as LINEAR measure



THE MEASUREMENT OF ANGLES

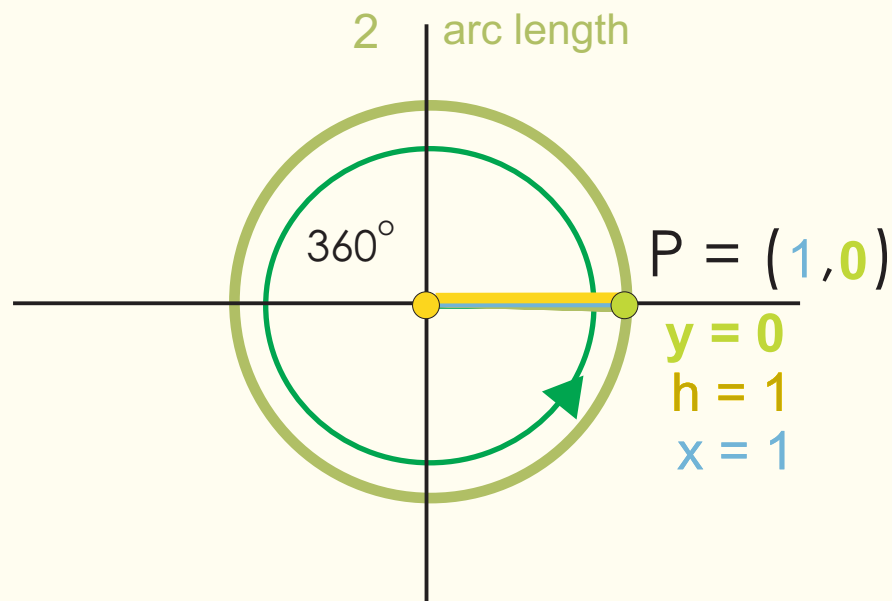
TRIGONOMETRIC MEASURE of 360°

$$\sin(360^\circ) := \frac{\text{opp}}{\text{hyp}} = \frac{0}{1}$$

$$\cos(360^\circ) := \frac{\text{adj}}{\text{hyp}} = \frac{1}{1}$$

$$\tan(360^\circ) := \frac{\text{opp}}{\text{adj}} = \frac{0}{1}$$

TRIANGULAR measure



RADIAN MEASURE of 360°

$$360^\circ = \quad \text{radians}$$

as LINEAR measure

